AH-1539 CV-19-S.E.

M.A./M.Sc. (Previous)

Term End Examination, 2019-20

Mathematics

Paper-I

Advanced Abstract Algebra

Time : Three Hours] [Maximum Marks: 100

**Note :** Answer any five questions. All Questions carry equal marks.

1- (a) If G is a finite group and Z is the centre then prove that the class equation of G is expressible as.

$$O(G)=o(z)+\frac{\Sigma}{a\notin Z}\{\frac{o(G)}{o(N(a))}\}$$

When the summation runs over the element a in each conjugate class.

(b) Prove that a normal subgroup H of a group G is maximal if and any it the group $\frac{G}{H}$ is simple.

2- (a) Prove that Enesy Hemomorphic image of a solvable group is solvable.

(b) Let N be a normal subgroup of a group G, if N and $\frac{G}{N}$ are solvable then prove that G is also. Solvable

3- (a) Prove that a group of order $P^n$(pis prime) is nilpotent.

(b) Prove that Enesy finite group has a composition series.

4- (a) If S be an ideal of a ring Z of all integers then prove that S is Maximal if and only if it is generated by some prime integers.

(b) Prove that the Submodull of the Quetient module $\frac{M}{N}$ are of the form $\frac{U}{N}$ where U is a sub module of M Containing N.

5- (a) Let R be a ring with unity. Show that an R module M is cyclic if and any if $M\cong\frac{R}{I}$ for some left ideal I of R.

(b) Define free module. If M be a free R module with a basis $\{e_1, e_2, e_3 ---- e_n\}$ Then prove $M\cong R^n$

6- (a) Prove that every finitely generated module is a Hemomorphic image of a finitely generated free module.